Statistics of the generalized maximum likelihood estimation in deformed exponential families

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An exponential family plays an important role in statistics and it is well-known that the maximum likelihood estimation can be geometrically explained in terms of information geometry. That is, an exponential family is a dually flat space with respect to the exponential and the mixture connections and the maximum likelihood estimator is obtained by the orthogonal projection of the mixture geodesic.
A deformed exponential family is a generalization of exponential families, which was originally introduced in the context of anomalous statistical physics [1]. From a viewpoint of information geometry, Matsuzoe and Henmi (2013) [2] showed that a deformed exponential family has two different kinds of dually flat structures as a statistical manifold. One of them is related with the U-divergence geometry in machine learning [3] and with robust statistics for the special case of q-exponential families. However, the statistical meaning of the other geometrical structure, which is a geometry of the generalized maximum likelihood estimation, is still not so clear although it seems to be quite natural from a geometrical point of view.
In this talk, we discuss a role and some properties of the generalized maximum likelihood estimation, which is defined by the maximization of the deformed log-likelihood function, in a deformed exponential family from a statistical point of view. Although there are some studies on it for an i.i.d. sample [4], [5], we especially focus on the case where there exists some correlation in the sample, which is implied by the generalized independence.

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